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Book review

The Dynamics of Patterns
by M.I. Rabinovich, A.B. Ezersky and P. Weidman (World Scientific, Singapore, 2000, 324 p.) ISBN 981 02 4056 2

Pattern formation is a wide domain of the science of systems driven far from equilibrium. The book presented by Rabinovich and colleagues is an interesting contribution to this field, mainly because it reviews most of the state of knowledge up to the publication date (August 2000) in systems, the dynamics of which is dominated by the emergence and subsequent evolution of the macroscopic ordering we call patterns. Within about three hundred pages, making a set of fourteen chapters and two appendices, we are invited to walk around in the realm of nonlinear dynamics of extended systems.

First few words to explain what “extended” means. Usually in systems driven far from equilibrium, states on the thermodynamic branch, those deriving continuously from the equilibrium state in the absence of driving, become unstable with respect to some mechanism that generate structures with a typical wavelength. For systems with dimensions of the order of this wavelength, called “confined,” the space coherence implied by the mechanism allows one to reduce the dynamics to a small subset of mode amplitudes, the evolution of which is described within the framework of dynamical systems with a small number of degrees of freedom. By contrast, the size of an “extended system” is large when compared to this unstable wavelength, at which “local” processes take place. The local time dynamics is governed by a conventional approach in terms of low dimensional dynamical systems which has to be further unfolded to account for space effects. Speaking of the dynamics of pattern thus refers to this more “global” point of view.

Appendix A entitled “A short guide to nonlinear dynamics” is devoted to a review of the background on dynamical systems of use primarily for confined systems but necessary to describe local processes in extended systems. The essentials of the theory, including chaos, are summarized but this is somehow misconcluded by a presentation of the problem of the transition to turbulence in convection that would have found a better place as a subsection of Appendix B. As a matter of fact, this second appendix, entitled “Key experiments in pattern formation,” presents several examples of instability mechanisms and, after the case of parametrically generated Faraday ripples, it indeed overviews two celebrated phenomena: convection patterns (Rayleigh–Bénard or Bénard–Marangoni), and chemical diffusion at the origin of Turing patterns or oscillatory instabilities (Belouzov–Zhabotinsky). The interesting case of thermocapillary waves is however missing, whereas it offers an ideal illustration of propagating dissipative waves well described by coupled Ginzburg–Landau equations with nonzero group velocity.

This being said about appendices, let us come back to the subject, i.e. patterns. For extended systems, a special approach to unfold the space dependence has to be undertaken, that makes reference to some underlying unbounded medium and its associated symmetries (translational, rotational, Galilean). Small scale coherence subsists but at large scales space–time disorder develops that remains to be interpreted. A kind of elimination of degrees of freedom different from that used for confined systems has to be developed, resting on multiple-scale analysis.

After a brief presentation of some patterns, regular or disordered, and the suggestion that they come from the superposition/interaction of several elementary “modes” in Chapter 1, Chapter 2 is devoted to few case studies of the linear stage of the dynamics leading to the emergence of these modes. In practice the ab initio study of realistic systems is usually so complicated that one has to turn to simplified models, the prototype of which is the Swift–Hohenberg (SH) model, serving as an illustration of nonlinear interaction processes and more technically as a testing ground for many theoretical approaches. Amplitude equations are next introduced as phenomenological models. A third level of description, phase equations, is then presented. Phase equations specifically account for the dynamics of nearly neutral modes associated with translational, rotational and Galilean invariances. All these topics are treated sketchily within the same chapter, without actual demonstration and, unfortunately, with sometimes imprecise quotation of original references.

Chapter 4 is next entirely devoted to a prototype of amplitude equations, the cubic complex Ginzburg–Landau equation. After a brief presentation of problems where it emerges quite naturally rather than as a model, some of its properties are described, which depend mostly on the competition between linear and nonlinear dispersion.

Emergence of order in the form of “crystals” and “quasi-crystals,” respectively issued from the interaction between “commensurate” or “incommensurate” modes is illustrated in Chapters 5 and 6, using models and amplitude equations introduced previously or examples from experiments on Faraday ripples.

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The breaking of order is the subject of the next three chapters. Since in extended systems, it is exceptional that the whole system be perfectly ordered, defects and inhomogeneities play an important role. Domain walls and dislocations in roll patterns and penta–hepta defects in hexagonal lattices first illustrate topological defects in the order-parameter field. Next, a different kind of wall is presented that separate competing states in multistable systems, with the extreme case of localized solutions understood as bound states of such wall solutions. Finally, the case of spirals is examined, with a stress on the difference between "active" spirals in oscillatory systems, such as those in the Ginzburg–Landau equation, and "passive" spirals in stationary but curved roll systems ideally forming target patterns. Chapter 12, devoted to the tools appropriate for measuring the amount of space disorder, is curiously separated from what precedes by two more anecdotal chapters. The first one, on patterns in soap films, deals with a system that seem to keep inescapable hydrodynamical features making it less universal, and the second one with patterning in colonies of microorganisms as a bridge towards the psychedelic views developed in the last chapter where hallucinations are interpreted in terms of electrical instabilities in the neuronal tissue; which goes to show that the nonlinear science of pattern dynamics could serve to something outside of physics or chemistry.

To conclude, the book presents itself as a well-documented review, owing to the breadth of the topics touched upon, but it can hardly be considered as an introduction to the field in spite of its appealing appearance. Indeed, illustrations are numerous and well chosen but results are most of the time given without explicit derivations or without reference to original work, some expressions are not defined (e.g., order parameter, convective instability. . .), some seminal remarks are misattributed (e.g., about variants of the SH model or the role of drift flows). Whereas this makes it specially appropriate as a guide or a reminder for a specialist in charge of lectures on the subject, clearly not much but something is missing about technical issues to make it suited for self-teaching (namely, a derivation of amplitude equations from the SH model and phase equations from amplitude equations). I acknowledge the fact that this idealistic aim is perhaps impossible to reach within a little more than 300 pages — at least is it much less technical and arid than the Rev. Mod. Phys. article by Cross and Hohenberg — and it is also the reason why I can freely recommend its reading by somebody with little prior knowledge of nonlinear dynamics but interested in a broad review of its applications to extended systems.

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